

New Massive Gravity Holography

U.Camara dS^{*1}, C.P.Constantinidis^{*2} and G.M.Sotkov^{*3}

^{*} Departamento de Física - CCE
Universidade Federal de Espirito Santo
29075-900, Vitoria - ES, Brazil

ABSTRACT

We investigate the holographic renormalization group flows and the classical phase transitions in two dimensional QFT model dual to the New Massive 3D Gravity coupled to scalar matter. Specific matter self-interactions generated by quadratic superpotential are considered. Assuming that the off-critical AdS_3/CFT_2 correspondence takes place, we reconstruct the exact form of the QFT_2 's β -function which allows to find the singular part of the reduced free energy. The corresponding scaling laws and critical exponents characterizing all the RG fixed points as well as the values of the mass gaps in the massive phases are obtained.

KEYWORDS: AdS/CFT, New Massive Gravity, Holographic RG Flows

¹e-mail: ulyssescamara@gmail.com

²e-mail: cpconstantinidis@gmail.com

³e-mail: sotkov@cce.ufes.br, gsotkov@yahoo.com.br

1 Introduction

The AdS_{d+1}/CFT_d correspondence [1] provides holographic description of $d = 4$ $SU(N)$ supersymmetric large N gauge theories and its off-critical (a) AdS_5/QCD_4 version is expected to solve the problem of the strong coupling regime of QCD_4 [2]. In this context the *two dimensional* case represents a rather "non-physical" problem, which however is known to be of *conceptual* importance. Since two dimensional (super) conformal group is infinite, the specific features of its unitary representations [3] allow to exactly calculate all the anomalous dimensions and the n -points correlation functions of all the primary and composite fields. Another important fact of purely *2D* nature is the existence of a vast variety of *integrable* perturbations of the corresponding CFT_2 's [4], as for example (super)sine-Gordon and the abelian affine (super) Toda models [5],[6], whose S -matrices, mass spectra, form-factors and some correlation functions are known exactly [7]. Apart from the practical use of all these *2D* models in the description of real condensed matter systems [8], the huge amount of available exact results also permits to realize non-trivial self-consistency checks of the (eventual) validity of the off-critical AdS_3/CFT_2 correspondence even *out* of its original superstring/supergravity/SUSY gauge theories frameworks.

In what concerns the lessons one can learn about the corresponding realistic higher dimensional $d = 4$ models, we should mention however one serious disadvantage when 3D Einstein gravity of negative cosmological constant is used as 3D "bulk" gravity. Since it has no local degrees of freedom its properties as well as the ones of its 2D dual are rather different from the properties of corresponding $d + 1 = 5$ versions. It is therefore interesting to study examples of the *off-critical* AdS_3/CFT_2 correspondence based on appropriate *extensions* of the Einstein 3D gravity, that have features similar to the ones of 4D and 5D Einstein gravity such as "propagating gravitons", non-trivial vacua solutions, etc. The simplest model of such extended 3D gravity is given by the following "higher derivatives" action, called New Massive Gravity (NMG)[9]:

$$S_{NMG}(g_{\mu\nu}, \sigma; \kappa, \Lambda) = \frac{1}{\kappa^2} \int d^3x \sqrt{-g} \left\{ \epsilon R + \frac{1}{m^2} \mathcal{K} - \kappa^2 \left(\frac{1}{2} |\vec{\nabla} \sigma|^2 + V(\sigma) \right) \right\} \quad (1)$$

$$\mathcal{K} = R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2, \quad \kappa^2 = 16\pi G, \quad \epsilon = \pm 1$$

It describes massive graviton (of two polarizations) interacting with scalar matter. One can consider the new \mathcal{K} terms above as one loop counter-terms appearing in the perturbative quantization of 3D Einstein gravity. As it was recently shown by Bergshoeff, Hohm and Townsend (BHT) [9] the above model, *unlike* the case of higher dimensional $D = 4$ and $D = 5$ Einstein gravities with one loop counter-terms added, turns out to be *unitary* consistent (ghost free) for the both choices $\epsilon = \pm 1$ of the "right" and "wrong" signs of the R -term, under certain restrictions on the values of the cosmological constant $\Lambda = -\frac{\kappa^2}{2} V(\sigma^*)$ and of the new mass scale m^2 .

The problem we are interested in concerns the *classical* critical phenomena that take place in the (euclidean) QFT_2 's dual to NMG model (1). More precisely we will describe the phase transitions that occur in 2D *classical statistical mechanics* models in infinite volume, whose thermodynamical limits represent models dual to NMG. According to the *off-critical* AdS_3/CFT_2 correspondence [10] the domain wall solutions (DW's) of 3D gravity models of negative cosmological constant provide an alternative "dual" description of the renormalization group (RG) flows in specific 2D *deformed* conformal field theories CFT_2 's. The models involved in the "boundary" QFT_2 part of this relation are usually realized as appropriate CFT_2 's (called $pCFT_2$'s) perturbed by marginal or/and relevant operators [2] that break conformal symmetry to its Poincare subgroup :

$$S_{pCFT_2}^{ren}(\sigma) = S_{CFT_2}^{UV} + \sigma(L_*) \int d^2x \Phi_\sigma(x^i) \quad (2)$$

The scale-radial duality [11] allows to further identify the “running” coupling constant $\sigma(L_*)$ of $pCFT_2$ with the scalar field $\sigma(z)$ and the RG scale L_* with the scale factor $e^{\varphi(z)}$ of DW’s solutions of the bulk gravity coupled to scalar matter as follows:

$$ds^2 = dz^2 + e^{\varphi(z)}(dx^2 + dt^2), \quad \sigma(x^i, z) \equiv \sigma(z), \quad L_* = l_{pl} e^{-\varphi/2} \quad (3)$$

Once the pair of dual theories is established, the set of “holographic rules” [11],[2],[12] allows to deduce many of the important features of the *quantum* $pCFT_2$ - as anomalous dimensions, fields expectation values, etc. - from the *classical* DW’s solutions of the corresponding “bulk” gravity.

The NMG’s vacuum and DW’s solutions, the unitarity conditions they have to satisfy and the values of the central charges of the conjectured dual CFT_2 ’s were extensively studied by different methods [9], [13],[14],[15],[16]. As is well known from the example of Einstein gravity the proper existence and the properties of the holographic RG flows in its 2D dual QFT strongly depend on the form of bulk matter interactions. If they permit DW’s solutions relating two unitary NMG vacua of different cosmological constants then we might have massless RG flows in the dual $pCFT_2$. However the construction of such solutions is a rather difficult problem and it requires the knowledge of an auxiliary function $W(\sigma)$ called superpotential that allows to reduce the corresponding DW’s gravity-matter equations to specific BPS-like I^{st} order system. The generalization of the superpotential method[16] to the case of NMG model (1) was recently introduced in refs.[14],[15] :

$$\kappa^2 V(\sigma) = 2(W')^2 \left(1 - \frac{\kappa^2 W^2}{2\epsilon m^2}\right)^2 - 2\epsilon \kappa^2 W^2 \left(1 - \frac{\kappa^2 W^2}{4\epsilon m^2}\right), \quad \dot{\varphi} = -2\epsilon \kappa W, \quad \dot{\sigma} = \frac{2}{\kappa} W' \left(1 - \frac{\kappa^2 W^2}{2\epsilon m^2}\right) \quad (4)$$

where $W'(\sigma) = \frac{dW}{d\sigma}$, $\dot{\sigma} = \frac{d\sigma}{dz}$ etc. It provides the explicit form of qualitatively new DW’s relating “old” to the “new” purely NMG vacua as well as of the corresponding $pCFT_2$ model’s β -function[15]. As it is shown in ref. [15] the simplest and most representative example that exhibits rich phase structure is the one generated by quadratic matter superpotential: $W(\sigma) = B\sigma^2 + D$. Some preliminary results concerning 3D gravitational origin of the phase transitions in this *three* scales $l_{pl} \ll L_{gr} < L_a = (\kappa D)^{-2}$ - model were presented in ref.[15].

The present paper is devoted to the complete description of the holographic RG flows and of the classical phase transitions in the $pCFT_2$ dual to the NMG model with quadratic matter superpotential. The critical exponents characterizing all the RG fixed points as well as the values of the mass gap in the massive phases are calculated.

2 CFT’s data of NMG model

Given the form of the superpotential $W(\sigma)$ and related to it I^{st} order system (4) that describes the radial evolution of the NMG’s scale factor and of the scalar field $\sigma(z)$. The scale-radial identifications (3) allow us to deduce the explicit form of the β -function of conjectured dual $pCFT_2$ [11],[12] in terms of the NMG’s superpotential:

$$\frac{d\sigma}{dl} = -\beta(\sigma) = \frac{2\epsilon}{\kappa^2} \frac{W'(\sigma)}{W(\sigma)} \left(1 - \frac{W^2(\sigma)\kappa^2}{2\epsilon m^2}\right), \quad l = \ln L_* \quad (5)$$

Let us briefly remind how one can extract the information about the critical properties of $pCFT_2$ model from eq.(5) and the way such CFT_2 data is related to the asymptotic behaviour of the NMG’s domain wall solutions [15] or equivalently to the shape of the matter potential $V(\sigma)$. The two types of real zeros of this β -function : (a) $W'(\sigma_a^*) = 0$ and (b) $W^2(\sigma_b^*) = \frac{2\epsilon m^2}{\kappa^2}$ indeed coincide with (part of) the extrema i.e. $V'(\sigma_A^*) = 0$ for $A = a, b$ of the matter potential $V(\sigma)$. Hence new

purely NMG i.e. type (b) critical points exist only in the case when $\epsilon m^2 > 0$. By construction both (a) and (b) critical points- describe AdS_3 vacua $(\sigma_A^*, \Lambda_{eff}^A)$ of the NMG model

$$ds^2 = dz^2 + e^{-2\epsilon\sqrt{|\Lambda_{eff}^A|}z}(dx^2 + dt^2), \quad A = a, b$$

where the effective cosmological constants Λ_{eff}^A are defined by the vacuum values of the corresponding scalar 3D curvature:

$$R = -2\ddot{\varphi} - \frac{3}{2}\dot{\varphi}^2 \equiv 8\epsilon(W')^2 \left(1 - \frac{\kappa^2 W^2}{2\epsilon m^2}\right) - 6\kappa^2 W^2 \quad (6)$$

i.e. we have $R_{vac} = -6\kappa^2 W^2(\sigma_A^*) = 6\Lambda_{eff}^A$. These critical points are known to correspond to II^{nd} order phase transitions occurring in $pCFT_2$ where it becomes conformal invariant. Therefore the critical behaviour of this 2D model is described by a set of CFT_2 's of central charges :

$$c_A = \frac{3\epsilon L_A}{2l_{pl}} \left(1 + \frac{L_{gr}^2}{L_A^2}\right), \quad L_{gr}^2 = \frac{1}{2\epsilon m^2} \gg l_{pl}^2, \quad \kappa^2 W^2(\sigma_A^*) = \frac{1}{L_A^2} \quad (7)$$

calculated in the approximation of small cosmological constants, i.e. $l_{pl} \ll L_{gr} < L_A$ by the Brown-Henneaux asymptotic method [17] appropriately adapted to the case of NMG coupled to scalar matter [16],[15].

It is natural to consider the *quantum* (euclidean) $pCFT_2$ in discussion as describing the universality class of the thermodynamical (TD) limit of certain 2D *classical* statistical models. We are interested in studying the *infinite volume* critical properties of these statistical models by using the Wilson's RG methods. As is well known (see for example [18], [5]) they are characterized by the scaling laws and the critical exponents of their TD potentials as for example the ones $y_A = \frac{1}{\nu_A}$ related to the singular part (s.p.) of the reduced free energy (per 2D volume) F_s^A , to correlation length ξ_A and to $\Phi_\sigma(x_i)$'s correlation functions:

$$F_s^A(\sigma) \approx (\sigma - \sigma_A^*)^{\frac{2}{y_A}}, \quad \xi_A \approx (\sigma - \sigma_A^*)^{-\frac{1}{y_A}}, \quad G_\Phi^A(x_{12}, \sigma) = \langle \Phi_\sigma(x_1) \Phi_\sigma(x_2) \rangle_A \approx \frac{e^{-\frac{|x_{12}|}{\xi_A}}}{|x_{12}|^{2(2-y_A)}} \quad (8)$$

at the neighbourhood of each critical point σ_A^* . Once the β -function (5) is given, it completely determines the scaling properties of TD potentials, correlation functions, etc. under infinitesimal RG transformations as follows [18]:

$$\begin{aligned} \beta(\sigma) \frac{dF_s(\sigma)}{d\sigma} + 2F_s(\sigma) &= 0, \quad \beta(\sigma) \frac{d\xi(\sigma)}{d\sigma} = \xi(\sigma), \\ |x_{12}| \frac{dG_\Phi(x_{12}, \sigma)}{d|x_{12}|} + \beta(\sigma) \frac{dG_\Phi(x_{12}, \sigma)}{d\sigma} + 2\left(2 + \frac{d\beta(\sigma)}{d\sigma}\right) G_\Phi(x_{12}, \sigma) &= 0 \end{aligned} \quad (9)$$

One can easily verify for example that the above critical exponents (related to the Φ_σ field scaling dimensions Δ_Φ^A) are given by the values of the β -functions derivatives:

$$y(\sigma_A^*) = 2 - \Delta_\Phi(\sigma_A^*) = -\frac{d\beta(\sigma)}{d\sigma} \Big|_{\sigma=\sigma_A^*} \quad (10)$$

In our case (5) they have the following explicit form (for $W \neq 0$)⁴:

$$y_a = y(\sigma_a^*) = \frac{2\epsilon W_a''}{\kappa^2 W_a} \left(1 - \frac{\kappa^2 W_a^2}{2\epsilon m^2}\right), \quad y_b = y(\sigma_b^*) = -\frac{4\epsilon(W_b')^2}{\kappa^2 W_b^2}, \quad W_b^2 = \frac{2\epsilon m^2}{\kappa^2} \quad (11)$$

⁴the *singular* points σ_s such that $W(\sigma_s) = 0$ (where β -function diverges) divide the coupling space in few independent regions

Their 3D-geometry counterparts appear in the asymptotics of the matter field $\sigma(z)$ of corresponding DW's solutions of NMG model (see ref.[15]) :

$$\sigma(z) \stackrel{z \rightarrow \pm\infty}{\approx} \sigma_A^* - \sigma_A^0 e^{\mp 2\Delta_A \sqrt{|\Lambda_{eff}^A|} z}, \quad \Delta_A = 1 + \sqrt{1 - \frac{m_\sigma^2(A)}{\Lambda_{eff}^A}}, \quad m_\sigma^2 = V''(\sigma_A^*) \quad (12)$$

thus confirming the basic rule of *AdS/CFT* correspondence[2]: the *scaling dimensions* of 2D fields are determined by the 3D effective *cosmological constants* Λ_{eff}^A and by the asymptotic σ -vacuum states⁵ *masses* $m_\sigma^2(\sigma_A^*)$ as follows:

$$m_\sigma^2(\sigma_A^*) = -\Lambda_{eff}^A y_A (y_A - 2) \quad (13)$$

Depending on the values of y_A (or equivalently of $m_\sigma^2(A)$) we can have three qualitatively different near-critical behaviours of the coupling constant $\sigma(l)$ and therefore different type of critical points determined by the dimensions of 2D fields Φ_σ . As is well known when $\Delta_\Phi < 2$ the corresponding *relevant* operator gives rise to an increasing RG flow away the (unstable) UV critical point, while for $\Delta_\Phi > 2$ the operator governing the flow is *irrelevant* and we observe decreasing RG flow towards the (stable) IR fixed point :

$$\begin{aligned} (UV) \quad & 0 < y_A < 2, \quad m_\sigma^2(A) < 0, \quad L_* \rightarrow 0, \quad \xi \rightarrow \infty, \quad e^\varphi \rightarrow \infty, \\ (IR) \quad & y_A < 0, \quad m_\sigma^2(A) > 0, \quad L_* \rightarrow \infty, \quad \xi \rightarrow 0, \quad e^\varphi \rightarrow 0 \end{aligned} \quad (14)$$

The "degenerate" case $y_A = 0$, i.e. of (asymptotically) massless matter $m_\sigma^2(A) = 0$, is known to describes marginal operators with $\Delta_\Phi = 2$. Such critical points correspond to *infinite order* phase transitions, characterized by an essential singularity $F_s^A(\sigma) \approx \exp\left(\frac{\mu_A}{\sigma - \sigma_A^*}\right)$ and $\xi_A \approx \exp\left(\frac{\rho_A}{\sigma - \sigma_A^*}\right)$ instead of the power-like scaling laws (8) for thermodynamic's potentials in the case of *IInd* order phase transitions. Negative $m_\sigma^2(A)$ (tachyons) for scalar fields in *AdS₃* backgrounds do not cause problems when the Breitenlohner-Freedman (BF) condition [21] :

$$\Lambda_{eff}^A \leq m_\sigma^2(A) \quad (15)$$

is satisfied. The unitarity of the purely gravitational sector of NMG model (1) requires that the following Bergshoeff-Hohm-Townsend (BHT) conditions [9]:

$$m^2 \left(\Lambda_{eff}^A - 2\epsilon m^2 \right) > 0, \quad \Lambda_{eff}^A \leq M_{gr}^2(A) = -\epsilon m^2 + \frac{1}{2} \Lambda_{eff}^A \quad (16)$$

to take place. They impose further restrictions on the values of the cosmological constant $\Lambda_{eff}^a = -\kappa^2 W_a = -\frac{1}{L_a^2}$ of type (a) critical points (i.e. on NMG vacua) :

$$0 \leq \frac{\kappa^2 W_a^2}{2\epsilon m^2} \leq 2, \quad \epsilon m^2 > 0 \quad (17)$$

and consequently on the central charges (7) of the corresponding CFT's. The type (b) NMG vacua is known to be always unitary [15] and whether it represents UV or IR critical point of the dual *pCFT₂* depends on the sign factor only : UV - for $\epsilon = -1$ since we have $y_b > 0$ and IR - for $\epsilon = 1$ case. The properties of the type (a) critical points (UV or IR) do depend on both - the sign of ϵ and on the particular form of the matter superpotential, as one can see from eq.(11).

⁵in the case of self-interactions the effective masses are defined around each of the extrema σ_A^* of $V(\sigma)$, i.e. $\sigma_A^{*\pm} = \sigma(z \rightarrow \pm\infty)$ and therefore we have $m_\sigma^2(\sigma_A^*) = V''(\sigma_A^*)$

3 Quadratic Superpotential CFT's

Let us consider the vacuum structure and related CFT_2 data of NMG model (1) with quadratic superpotential $W(\sigma) = B\sigma^2 + D$, introduced in ref.[15], where its DW's solutions have been found. It represents the simplest example of extended 3D gravity, whose holographically dual $pCFT_2$ model still permits rather explicit description and as we shall see it exhibits rich spectrum of different critical phenomena. Its β -function (5) is parametrized by five parameters (B , D , m^2 , ϵ , $\kappa^2 = 16\pi l_{pl}$) - the same that determine the shape of the matter potential $V(\sigma)$ according to eq.(4). It is important to remember that the classification of the qualitatively different solutions of the RG eq.(5) that describe different critical behaviours of the corresponding 2D dual models requires the complete specification of the qualitatively different regions of the above mentioned parameter space, namely the number and the type of the RG critical points in function of the values of superpotential's parameters. Independently on the values of the parameters B and D we always have one type (a) vacuum $\sigma_a^* = 0$ represented by AdS_3 of cosmological constant $\Lambda_{eff}^a(\sigma_a^*) = -\kappa^2 D^2$. The $CFT_2(a)$ describing this critical point has central charge given by eq.(7) with $L_A^2 = \frac{1}{\kappa^2 D^2}$. The corresponding scaling dimension y_a of $\Phi_\sigma^{(a)}$ has the form:

$$y_a = \frac{4\epsilon B}{D\kappa^2} \left(1 - \frac{D^2 \kappa^2}{2\epsilon m^2}\right) \quad (18)$$

and it can be positive (UV - CFT_2) or negative (IR - CFT_2) depending on the values of ϵ , B and $D \neq 0$ ⁶.

We choose to further investigate the particular case of $\epsilon m^2 > 0$ only, where we can have in principle few type (b) critical points. We next fix the sign of $B > 0$. Then the available type (b) RG fixed points, determined by the real roots of equation $W^2(\sigma_b^*) = \frac{2\epsilon m^2}{\kappa^2}$ are given by:

$$(\sigma_\pm^*)^2 = \pm \frac{\sqrt{2\epsilon m^2}}{\kappa B} - \frac{D}{B}, \quad (\sigma_-^*)^2 \leq (\sigma_+^*)^2 \quad (19)$$

Note that there exist two *critical values* of D :

$$D_{cr}^\pm = \pm D_{cr}, \quad D_{cr} = \frac{\sqrt{2\epsilon m^2}}{\kappa} = \frac{1}{\kappa L_{gr}} \quad (20)$$

for which two of the (b) vacua : $\pm|\sigma_+^*|$ or $\pm|\sigma_-^*|$ coincide with the (a) one $\sigma_a = 0$, giving rise to an inflection (i.e.massless) point $V''(\sigma_a) = 0$ of the matter potential. It is then clear that depending on the values of D we have to distinguish the following three regions in the parameters space : (1) *no one* type (b) vacuum for $D > D_{cr}$; (2) *two* type (b) vacua $\{\pm|\sigma_+^*|\}$ for $-D_{cr} < D < D_{cr}$ and (3) *four* type (b) vacua $\{\pm|\sigma_+^*|, \pm|\sigma_-^*|\}$ for $D < -D_{cr}$. Remembering the definitions of the two NMG scales $L_a^2 = \frac{1}{\kappa^2 D^2}$ and L_{gr}^2 , we realize that the above division of the parameters space of regions of different number of critical points is in fact determined by the relations between these scales: $L_a^2 < L_{gr}^2$ for region (2); $L_a = L_{gr}$ on the borders (2)-(1) and (2)-(3); $L_a^2 > L_{gr}^2$ for both regions (1) and (3). Each one of these regions describe qualitatively different critical properties of the corresponding dual $pCFT_2$, governed by the different shapes of the NMG matter potential.

By definition the CFT_2 's describing all the type (b) critical points have equal central charges $c_b = \frac{3L_{gr}}{l_{pl}}$ and the dimensions $y_\pm = y_b(\sigma_b^*)$ of the corresponding dual 2D fields are given by:

$$y_\pm = \frac{16\epsilon B}{\kappa \sqrt{2\epsilon m^2}} \left(\frac{D\kappa}{\sqrt{2\epsilon m^2}} \mp 1 \right) \quad (21)$$

⁶we exclude the particular case $D = 0$ that corresponds to flat M_3 NMG vacua ,i.e. $\Lambda_a(D = 0) = 0$ and CFT_2 of $c_a(D = 0) = \infty$, which is not interesting in the AdS/CFT context.

Again as in the case of type (a) CFT_2 , the y_{\pm} signs are determined by ϵ and the superpotential parameters and depending on the regions (1) to (3) σ_b^* can describe UV or IR critical points of second order phase transitions. Observe however the drastic changes that occur at $D = \pm D_{cr}$, i.e. on the borders between the regions. The fact that we have now $y_a(\pm D_{cr}) = 0$ serves as an indication that such critical point describes *new* type (of infinite order) phase transition, as one can see by comparing the forms of the corresponding solutions $\sigma = \sigma(l, D)$ of RG equations (5),(9):

$$\begin{aligned}\xi(D = \pm D_{cr}) &\approx e^{-l} \approx e^{\frac{\varphi}{2}} = \left(1 \pm \frac{2\sqrt{2\epsilon m^2}}{\kappa B \sigma^2}\right)^{\frac{1}{y_{\mp}(cr)}} e^{\frac{\rho_0}{\sigma^2} - \frac{2}{y_{\mp}(cr)}}, \quad y_{\mp}(cr) = \pm \frac{32\epsilon B}{\kappa \sqrt{2\epsilon m^2}} \\ \xi(D \neq \pm D_{cr}) &\approx e^{-l} \approx e^{\frac{\varphi}{2}} = (\sigma^2)^{-\frac{1}{2y_0}} \left((\sigma_+^*)^2 - \sigma^2\right)^{-\frac{1}{y_+}} \left((\sigma_-^*)^2 - \sigma^2\right)^{-\frac{1}{y_-}}\end{aligned}\quad (22)$$

where we have denoted by $\rho_0 = -\frac{m^2}{8B^2}$ the critical index of "marginal" point $\sigma_0^* = 0$. Note that the specific power-like singularity for $D \neq \pm D_{cr}$ becomes essential singularity when $D = \pm D_{cr}$.

The above discussion makes clear that the description of the critical properties of $pCFT_2$ model with β -function given by (5) for each one of the regions (1), (2) and (3) requires the knowledge of an (ordered) set (c_A, y_A) of well defined CFT_2 's corresponding to the RG fixed points σ_A^* . Together with two singular points $\sigma_s^2 = -\frac{D}{B}$ (that exist for $D < 0$ only) they divide the coupling space $\sigma \in R$ in few intervals, say $\sigma \in (|\sigma_s|, |\sigma_+^*|_{UV})$, or $(0_{IR}, |\sigma_+^*|_{UV})$, etc. to be recognized as different phases of $pCFT_2$ model. For example, in *region* (2) we have to consider separately the case (2+) when $D \in (0, D_{cr})$ of three critical points only:

$$(2+) \quad \epsilon = -1: \quad (-\infty, -|\sigma_+^*|_{UV}) \quad (-|\sigma_+^*|_{UV}, 0_{IR}) \quad (0_{IR}, |\sigma_+^*|_{UV}) \quad (|\sigma_+^*|_{UV}, \infty) \quad (23)$$

describing four (2+)-phases, from the (2-) one of $D \in (-D_{cr}, 0)$, where we also have two (intermediate) singular points:

$$(2-) \quad \epsilon = -1: \quad ||(-\infty, -|\sigma_+^*|_{UV}) \quad (-|\sigma_+^*|_{UV}, -|\sigma_s|)||, \quad ||(-|\sigma_s|, 0_{UV}) \quad (0_{UV}, |\sigma_s|)||, \\ ||(|\sigma_s|, |\sigma_+^*|_{UV}) \quad (|\sigma_+^*|_{UV}, \infty)|| \quad (24)$$

They separate the coupling space in six different (2-) phases organized in three disconnected regions denoted by $||...||$. The index UV or IR, say 0_{IR} or $|\sigma_+^*|_{UV}$, marks the type of the RG fixed points (related to the sign of y_A) for $\epsilon = -1$ and $m^2 < 0$. In the case of $\epsilon = 1$ and $m^2 > 0$ the corresponding (2 \pm)-phase structure is identical to the above one, but now with *UV* and *IR* *interchanged*. Note the important difference between the regions (2+) and (2-) : all the critical points in region (2-) are of UV type (for $\epsilon = -1$) and all of IR type when $\epsilon = 1$, while in (2+) we have both UV and IR critical points. This fact reflects *different* asymptotic properties of the DW's in regions (2 \pm) representing (a) AdS_3 spaces of two boundaries (Janus-type) in the case (2-) and of one boundary-one horizon in the (2+) case [15]. Their $pCFT_2$ counterparts turns out to describe qualitatively different RG flows - massive in (2-) UV-UV intervals and massless in the (2+) UV-IR case.

It is worthwhile to mention the Z_2 symmetry $\sigma \rightarrow -\sigma$ of the NMG model with quadratic superpotential and of the RG equations (5) as well. As a consequence the phase structure in all of the regions remains invariant under σ reflections. Therefore it is enough to study just the half of the phases, say ones corresponding to $\sigma \geq 0$. We next consider *region* (3) where we have *five* critical points and two singular ones. The coupling space is now divided in three disconnected parts, containing eight phases. For $\sigma > 0$ we find the following four phases:

$$(3) \quad \epsilon = -1: \quad (0_{IR}, |\sigma_-^*|_{UV}) \quad (|\sigma_-^*|_{UV}, |\sigma_s|), \quad (|\sigma_s|, |\sigma_+^*|_{UV}) \quad (|\sigma_+^*|_{UV}, \infty). \quad (25)$$

Similarly, we realize that *region* (1) is formed by two phases $(-\infty, 0_{UV})$ and $(0_{UV}, \infty)$ only. The phase structure of $pCFT_2$ model corresponding to the two "borders" $D = \pm D_{cr}$ between the regions has the following symbolic form for $\epsilon = -1$ and $D = D_{cr}$: $(-\infty, 0_{mar})$, $(0_{mar}, \infty)$, while for $D = -D_{cr}$ we find six phases. Here we list its phase structure for positive σ only: $(0_{mar}, |\sigma_s^{cr}|)$, $(|\sigma_s^{cr}|, |\sigma_+^{cr}|_{UV})$, $(|\sigma_+^{cr}|_{UV}, \infty)$, where 0_{mar} denotes the critical point of infinite order characterized by $y_0 = 0$ (i.e. Φ_σ is marginal). The $|\sigma_+^{cr}|_{UV} = \sqrt{\frac{2D_{cr}}{B}}$ represent UV fixed RG point defined by the presence of relevant operator of dimension $\Delta_\mp = 2 - y_\mp(cr) < 2$. Note that all these critical points are described by CFT_2 's of equal central charges $c_{cr} = \frac{3L_{gr}}{l_{pl}}$.

Few comments about the dependence of the properties of the solutions of the RG eq.(5) on the values of the parameter B are in order. Let us remind that the above described "phase" structure of the parameters space was derived under the condition $B > 0$ and varying the remaining parameter D of the superpotential. It is straightforward to verify that the corresponding results for *negative* values of B can be obtained from the ones of *positive* values of B (as above) by applying the following formal rules

$$B \rightarrow -B: \quad \sigma_+^* \rightarrow \sigma_-^*, \quad reg.(3) \rightarrow reg.(1), \quad reg.(2+) \rightarrow reg.(2-), \quad \epsilon \rightarrow -\epsilon \quad (26)$$

and *without* changing the index UV or IR of the critical points.

4 Holographic RG flows and phase transitions

The CFT_2 's data $(\sigma_n^*, c_n, y_n)_{UV/IR}$ specific for each parameters space region, established in Sect.3. above, provide the boundary conditions necessary for the derivation of the solutions of RG eqs.(5) and(9) characterizing each phase $p_{nk} = (\sigma_n^*, \sigma_k^*)$. The RG flows by definition represent the way the coupling constant $\sigma(l, D)$ is running between two neighbour critical points when the RG scale L_* increases from $L_*^{UV} = 0$ (i.e. $l_{UV} = \infty$) to $L_*^{IR} = \infty$ (i.e. $l_{IR} = -\infty$). Depending on the behaviour of the correlation length $\xi(\sigma)$, the s.p. of the free energy $F_s(\sigma)$ (and its derivatives) and of the correlation functions $G_\Phi(x_{12}, \sigma)$ we distinguish in the non-degenerate case $D \neq \pm D_{cr}$ the following three types of phases:

$$\begin{aligned} (1) \quad & massless(UV/IR) : 0 < L_* \leq \infty \quad \xi(\sigma_{UV}^*) \approx \infty, \xi(\sigma_{IR}^*) \approx 0; \sigma(-\infty) = \sigma_{UV}^*, \sigma(\infty) = \sigma_{IR}^*, \\ (2) \quad & massive(UV/\infty) : 0 < L_* \leq L_*^{ms} \quad \xi(\sigma_{UV}^*) \approx \infty, \quad \xi(\sigma \approx \infty) \approx L_*^{ms}, \\ (3) \quad & Janus(UV_+/\sigma_s/UV_-) : 0 < L_* \leq L_*^{max} \quad \xi(\sigma_{UV}^\pm) \approx \infty, \quad \xi(\sigma_s) \approx L_*^{max} \end{aligned} \quad (27)$$

The simplest example is provided by the phase structure of $pCFT_2$ model in region (2+) [15]. For $\sigma > 0$ and $\epsilon = -1$ it contains two phases: $p_{ml} = (0_R, |\sigma_+^*|_{UV})$ and $p_{ms} = (|\sigma_+^*|_{UV}, \infty)$, characterized by the singularities and asymptotic behaviour of the solutions of eqs.(5),(9):

$$\xi_{(2+)}(\sigma, \sigma_0) \approx e^{-l} = \left(\frac{\sigma^2}{\sigma_0^2} \right)^{-\frac{1}{2y_0}} \left(\frac{(\sigma_+^*)^2 - \sigma^2}{(\sigma_+^*)^2 - \sigma_0^2} \right)^{-\frac{1}{y_+}} \left(\frac{(|\sigma_-^*|^2 + \sigma^2)}{(|\sigma_-^*|^2 + \sigma_0^2)} \right)^{-\frac{1}{y_-}} \quad (28)$$

where $\sigma_0 = \sigma(l=0)$ represents the "initial" condition of RG rescalings, i.e. $L_*^{(0)} \approx 1$. We therefore recognize the $p_{ml} = (0_R, |\sigma_+^*|_{UV})$ -phase as describing *massless* RG flow starting from UV critical point $|\sigma_+^*|_{UV}$ where $\xi_{(2+)}(\sigma_+^*|_{UV}, \sigma_0) \approx \infty$ (i.e. $L_*^{UV} \approx 0$) and terminating at IR one 0_{IR} where as expected we have $\xi_{(2+)}(0_{IR}, \sigma_0) \approx 0$ and $L_*^{IR} \approx \infty$. Note that in the case ($\epsilon = 1, m^2 > 0$) the direction of the flow is inverted, since as we have explained in sect.3. now $\sigma = 0$ becomes of UV

type and $\sigma = |\sigma_+^*|$ of IR one. Although we have no characteristic (mass) scale in this interval $\sigma \in p_{ml} = (0_R, |\sigma_+^*|_{UV})$, our $pCFT_2$ model however is *not* conformal invariant.

The (2+)-phase corresponding to the coupling space interval $\sigma \in p_{ms} = (|\sigma_+^*|_{UV}, \infty)$ is characterized by the *finite* value of correlation length for $\sigma \rightarrow \infty$:

$$\xi_{(2+)}(\sigma \rightarrow \infty, \sigma_0 > |\sigma_+^*|_{UV}) \approx e^{-l_{ms}} = (\sigma_0^2)^{\frac{1}{2y_0}} (\sigma_0^2 - (\sigma_+^*)^2)^{\frac{1}{y_+}} (|(\sigma_-^*)|^2 + \sigma_0^2)^{\frac{1}{y_-}} \quad (29)$$

as one can easily verify from the limit of eq.(28) taking into account the remarkable "resonance" property $\frac{1}{2y_0} + \frac{1}{y_+} + \frac{1}{y_-} = 0$, specific for our quadratic superpotential. We observe that in this phase the coupling constant runs to infinity while the RG scale is running in the finite interval $L_* \in (0, L_*^{(ms)})$ thus defining particular *mass gap*

$$M_{(ms)} \approx \frac{1}{L_*^{(ms)}} = (\sigma_0^2)^{-\frac{1}{2y_0}} (\sigma_0^2 - (\sigma_+^*)^2)^{-\frac{1}{y_+}} (|(\sigma_-^*)|^2 + \sigma_0^2)^{-\frac{1}{y_-}} \quad (30)$$

As a consequence the corresponding Φ_σ correlation function (9) changes its behaviour including now at the leading order specific exponential decay term $e^{-M_{ms}|x_{12}|}$ that determines the massive properties of this $pCFT_2$ -phase. We have therefore an example of phase transition from massless to the massive phase that occurs at the UV critical point $|\sigma_+^*|$ in the (2+)-phase of $pCFT_2$ model. The 3D gravity description of such phase transition involves *two different* NMG solutions having coinciding boundary conditions $(|\sigma_+^*|, \Lambda_{eff}^+, \Delta_+)$ at their common boundary $z \rightarrow \infty$, i.e. at $\sigma(\infty) = |\sigma_+^*|$. The massive phase is "holographically" described by *singular* DW metrics giving rise to (a) AdS_3 space-time with naked singularity [15], while the massless one corresponds to the regular DW (constructed in ref.[15]) interpolating between the two NMG vacua $|\sigma_+^*|_{UV}$ and 0_R .

The above analysis of the critical phenomena in $pCFT_2$ model (and their 3D geometrical counterparts) based on the standard statistical mechanical and RG methods, allows us to establish the *basic rule* of the off-critical (a) AdS_3/CFT_2 correspondence, namely: the NMG-geometrical description of the *phase transitions* in its dual $pCFT_2$ model is given by the analytic properties - poles, zeros, cuts and essential singularities - of the *scale factor* e^φ of 3D DW's metrics of the (a) H_3 (euclidean) type:

$$F_s^{(2+)}(\sigma, \sigma_0) \approx e^{2l} \approx \xi_{(2+)}^{-2} \approx e^{-\varphi(\sigma)} \quad (31)$$

as a function of the matter field σ obtained by excluding the radial variable z from the corresponding DW's solutions[15]. Another important ingredient of the *off-critical holography* is the so called Zamolodchikov's central function for NMG model ⁷ introduced in refs.[23],[15] :

$$C(\sigma) = -\frac{3}{2G\kappa W(\sigma)} \left(1 + \frac{\kappa^2 W^2(\sigma)}{2\epsilon m^2} \right) \quad (32)$$

which at the critical points $\sigma_{A\pm}^*$ takes the values (7). Remember that according to the I^{st} order eqs.(4) we have $W(\sigma) = -\frac{\dot{\varphi}}{2\epsilon\kappa}$ and therefore the central charges c_A and the central function as well are geometrically described by the log-derivative $\dot{\varphi}$ of the scale factor. As a consequence of its definition (32) and of the RG eqs. (5) we conclude that [15]:

$$\frac{dC(\sigma)}{dl} = -\frac{3}{4GW(\sigma)} \left(\frac{d\sigma}{dl} \right)^2 \quad (33)$$

⁷It represents a natural generalization [23] of the well known result for $m^2 \rightarrow \infty$ limit [12],[11]

Hence for $W(\sigma)$ positive (as in our example) the central function is decreasing during the massless flow, i.e. we have $c(|\sigma_+^*|_{UV}) > c(0_{IR})$ for $\epsilon = -1$.

The RG flows in region (2-) are rather different from the ones of (2+) due to the fact that all the critical points are now of UV-type and to the presence of singular points as one can see from eq.(24). The massive phase $(|\sigma_+^*|_{UV}, \infty)$ coincides with the corresponding one in reg.(2+) and the mass gap is given again by M_{ms} of eq.(30) except that the values of the exponents $y_{\pm} > 0$ and $y_0 > 0$ are different due to negative sign of $D < 0$ in this region. The new massive phase is related to the Janus type DW's solutions (see ref.[15]) connecting two critical points (NMG vacua) with singular point in between, i.e. $0_{UV}/|\sigma_s|/|\sigma_+^*|_{UV}$ both provided with *relevant* operators. As one can see from the scale factor and from the correlation length $\xi_{(2-)}$ behaviours (36) the RG scale is now start running from $L_* = 0$ at the both 0_{UV} and $|\sigma_+^*|_{UV}$ critical points and it gets its maximal value $L_*^{(max)}$ at the "end point" $|\sigma_s| = \sqrt{\frac{|D|}{B}}$. Note the important difference with the normal massive phase where the $L_*^{(ms)}$ was reached for $\sigma \approx \infty$. The proper existence of $L_*^{(max)}$ however introduces mass scale :

$$M_J^{(2-)}(\sigma_s, \sigma_0) \approx e^{l^{max}} = \left(\frac{\sigma_s^2}{\sigma_0^2}\right)^{\frac{1}{2y_0}} \left(\frac{(\sigma_+^*)^2 - \sigma_s^2}{(\sigma_+^*)^2 - \sigma_0^2}\right)^{\frac{1}{y_+}} \left(\frac{(|\sigma_-^*|)^2 + \sigma_s^2}{(|\sigma_-^*|)^2 + \sigma_0^2}\right)^{\frac{1}{y_-}} \quad (34)$$

specific for the new Janus-massive phase. Hence in this case we have two different massive phases that start from the same critical point $|\sigma_+^*|_{UV}$. This massive-to-massive phase transition is characterized by the ratio of the two mass gaps:

$$\frac{M_J^{(2-)}}{M_{(ms)}^{(2-)}} = \left(\sigma_s^2\right)^{\frac{1}{2y_0}} \left((\sigma_+^*)^2 - \sigma_s^2\right)^{\frac{1}{y_+}} \left(|(\sigma_-^*)|^2 + \sigma_s^2\right)^{\frac{1}{y_-}} \quad (35)$$

which differently from the corresponding ξ 's and mass gaps is completely determined by the superpotential data and turns out to be an important characteristics of the $pCFT_2$ model. The NMG description of (2-)- phase diagram is given now by one Janus-type DW and one singular solution representing naked-singularity.

The phase structure of $pCFT_2$ model in region (3) turns out to combine all the critical phenomena we have observed in regions (2 \pm). Consider again the $\sigma > 0$ case. The coupling space is now divided in four intervals(i.e.phases): $(0_{IR}, |\sigma_-^*|_{UV})$, $(|\sigma_-^*|_{UV}, |\sigma_s|)$, $(|\sigma_s|, |\sigma_+^*|_{UV})$, $(|\sigma_+^*|_{UV}, \infty)$ containing three critical and one singular points. As one can verify from the behaviour of the corresponding correlation length:

$$\xi_{(3)}(\sigma, \sigma_0) \approx \left(\frac{\sigma^2}{\sigma_0^2}\right)^{-\frac{1}{2y_0}} \left(\frac{(\sigma_+^*)^2 - \sigma^2}{(\sigma_+^*)^2 - \sigma_0^2}\right)^{-\frac{1}{y_+}} \left(\frac{(|\sigma_-^*|)^2 - \sigma^2}{(|\sigma_-^*|)^2 - \sigma_0^2}\right)^{-\frac{1}{y_-}} \quad (36)$$

that the phase $(0_{IR}, |\sigma_-^*|_{UV})$ is describing massless RG flow similar to the one in the region (2+) but involving the new critical point $|\sigma_-^*|_{UV}$. The next two phases $(|\sigma_-^*|_{UV}, |\sigma_s|)$ and $(|\sigma_s|, |\sigma_+^*|_{UV})$ are both representing Janus -massive phases, while the last one $(|\sigma_+^*|_{UV}, \infty)$ is identical to the mass phase of region (2+) except that the exponents y_A with $A = \pm, 0$ as well as the mass gap formula are the ones specific for the region (3) with $D < -D_{cr}$. It is evident that the holographic description of the reg.(3) phase structure in terms of NMG's DW solutions consists of three different DW's of common boundaries : one of UV-IR type interpolating between two different NMG vacua of cosmological constants Λ_0^{eff} and Λ_-^{eff} , one of Janus type and the last one that involves naked singularity.

The nature of the phase transitions in $pCFT_2$ occurring at the borders of the parameter space $D = \pm D_{cr}$ is rather different of the ones of II^{nd} order we have described above. The essential singularity of $\xi(\sigma, D_{cr})$ and $F_s^{cr}(\sigma)$ at the infinite order phase transition critical point $\sigma = 0_{mar}$ is the only RG fixed point we have in the case $D = D_{cr}$. The corresponding massive phase $(0_{mar}, \infty)$ is characterized by the mass gap $M_{ms}^{cr} \approx e^{\frac{2}{y_{\mp}(cr)}}$ obtained from the $\sigma \rightarrow \infty$ limit of eq.(22). The phase structure in the case $D = -D_{cr}$ is richer: for $\sigma > 0$ we have one "marginal" critical point 0_{mar} , one singular point $|\sigma_s^{cr}|$ and one UV critical point $|\sigma_+^{cr}|_{UV}$ giving rise to four *massive* phases. The first two massive phases $(0_{mar}, |\sigma_s^{cr}|)$ and $(|\sigma_s^{cr}|, |\sigma_+^{cr}|_{UV})$ are of Janus type of mass gap $M_J^{cr} \approx e^{\frac{4}{y_{\mp}(cr)}} = (M_{ms}^{cr})^2$ and the last one $(|\sigma_+^{cr}|_{UV}, \infty)$ is the standard strong coupling massive phase $(|\sigma_+^{cr}|_{UV}, \infty)$. The phase transitions at $\sigma = |\sigma_+^{cr}|_{UV}$ point is of J-massive-to-massive (weak-to-strong coupling) type, quite similar to the one observed in the region (2-) above (i.e. of second order), but indeed with different mass ratio $\frac{M_J^{cr}}{M_{ms}^{cr}} = e^{\frac{2}{y_{\mp}(cr)}}$.

5 Discussion

Our investigation of the *classical* critical phenomena in the $pCFT_2$'s duals to the NMG models with quadratic matter superpotential has revealed many essential features of these 2D non-conformal models leaving however still open the problem of their complete identification. It is important to emphasize that the phase transitions we have described concern the TD limits of certain 2D *classical* statistical models (s.m.), related to the $pCFT_2$ in discussion. We have studied the *infinite volume* critical properties of these statistical models by using the Wilson RG methods. As it well known (see sect.4.5. of the Cardy's book[18]) the *finite* temperature phase transitions in the classical $d = 2$ s.m. in infinite volume correspond to *zero* temperature phase transition in certain equivalent quantum $d = 1$ s.m.(or its TD 1 + 1 QFT limit) when some other coupling in the quantum model becomes critical, say the transverse magnetic field in the case of 1D Ising model. Observe that the temperatures used in both models are different: the inverse temperature (i.e. $\frac{1}{kT_1}$) in the quantum 1D model corresponds to the period of the extra (time) direction, while the temperature in 2D classical s.m. is related to the extra coupling constant in 1D model. Hence the description of finite T_1 -phase transitions in the quantum 1D s.m. requires to study the finite-size effects in its 2D classical counterpart. This fact explains the successful use of DW solutions in the descriptions of classical phase transitions (in 2D s.m.) instead of say black holes and other periodic (or finite time) solutions of NMG model, which are indeed the geometric ingredients required in the investigation of finite T_1 quantum phase transitions.

The detailed description of the main features - critical exponents, mass gaps, s.p. of the reduced free energy - of the variety of second and infinite order classical phase transitions in 2D s.m. models that are *conjectured* to be dual of the NMG model (1), has led us to the following important rule of the off-critical AdS_3/CFT_2 correspondence: the *phase transitions* observed in the dual $pCFT_2$ models are determined by the analytic properties of the *scale factor* e^φ of 3D (euclidean) DW's type metrics of NMG model [15] written as a function of the matter field σ . As we have shown by using RG methods, the inverse of the *scale factor* is proportional of the s.p. of the *free energy*. In order to calculate the exact values of the entropy, the specific heat and other important TD characteristics one need to know the finite part of the free energy as well, which is a rather complicated problem even for the simplest 2D s.m. models. It is well known however that the infinite 2D conformal symmetry at the *critical points* offers powerful methods, based on the knowledge of the characters of the Virasoro algebra representations, which allow to construct the exact form of the corresponding

"critical" partition functions. Let us remind once more that all the information about holographic RG flows and phase transitions in the QFT_2 duals to the NMG model (1) we have extracted from the I^{st} order eqs. (4) is not sufficient for the complete identification of the $pCFT_2$ dual of NMG. One has to further consider the difficult problem of the construction of the off-critical correlation functions of 2D fields dual to 3D matter scalar by applying AdS/CFT methods[2] , [10] and to next compare with the known results of corresponding 2D models[5],[7].

Another important problem concerning the $(a)AdS_3/pCFT_2$ correspondence in the particular case of NMG model (1), is related to the *negative values* of the central charges (7) for $\epsilon = -1$ and $m^2 < 0$, that are usually interpreted as *non-unitary* CFT_2 's. Let us assume that all these CFT_2 's (without any extra symmetries present) are described by the representations of two commuting Virasoro algebras, characterized by their central charges $c_L = c_R = c$ and the set of scaling dimensions and spins $(\Delta_{(a)}, \bar{\Delta}_{(a)})$. The allowed values of c are usually divided in four intervals: (i) $c < 0$; (ii) $0 < c < 1$; (iii) $1 < c < 25$ and (iv) $c > 25$. The case (iii) is excluded from the considerations since it leads to complex values for the scaling dimensions and to non-unitary representations. In all the cases when $c < 0$ the corresponding CFT_2 's contain primary fields (states) of negative dimensions (and negative norms) and hence they represent non-unitary QFT_2 's as well⁸. As is well known in the interval $0 < c < 1$ there exists an infinite (discrete) series of "minimal" unitary models (m.m.) corresponding to $c_-^{(p)} = 1 - 6Q_p^2$ with $Q_p = \sqrt{\frac{p+1}{p}} - \sqrt{\frac{p}{p+1}}$ and $p = 3, 4, 5, \dots$. Finally for $c > 25$ one has unitary representations that are used in the quantization of the Liouville model [25]: $c_+(b) = 1 + 6Q_b^2$ with $Q_b = b + \frac{1}{b}$, where the parameter b is related to the Liouville coupling constant. On the other hand the derivation of the Brown-Henneaux [17] central charge formula $c = \frac{3L}{2G}$ as well as its NMG generalizations (7) are based on the "Dirac quantization" of the classical Poisson brackets Virasoro algebra representing particular bulk diffeomorphisms preserving the asymptotic form of the boundary metrics and by further identifying the classical central charge c_{class} for $L \gg l_{pl}$ with the "quantum" central charge of the "dual" boundary CFT_2 . The well known fact coming from the standard procedure of the Liouville model's [25] and of the "minimal" models quantizations [24] is that the central charge is receiving quantum corrections as for example from $c_{class}^+ = 1 + 6b^2$ to $c_q^+ = 1 + 6(b + \frac{1}{b})^2$. As it is shown in refs.[24] the *classical* (Poisson brackets) Virasoro algebra of central charge $c_{class}^- = 6Q_{cl}^2$ generated by L_n^{cl} give rise to the *quantum* one generated by $\tilde{L}_n = \hbar L_n^{cl}$ of central charge $c_q^- = 1 - 6\frac{Q_{cl}^2}{\hbar}$. In the classical limit $\hbar \rightarrow 0$ one obtains $c_q \rightarrow c_{cl} \approx -\infty$, i.e. the corresponding classical (and semiclassical) central charges are very big *negative* numbers. Similarly for the limits of Liouville's model[25] central charges we have $c_{cl}^+ \approx \infty$. Hence the classical (and semi-classical) large negative central charges are common feature for all the $c_q^- < 1$ models and of their supersymmetric $N = 1$ extensions. Further investigations of the limiting properties of the dimensions of the primary fields (and of their correlation functions) are needed in order to conclude whether such 2D CFT's belong to the non-unitary ($c_q < 0$) case or else to the interval $0 < c_q < 1$, where unitary models are known to exist. For the case of large positive classical central charges, unitarity requires that $c_q > 25$. It is important to note that the above considerations are valid in the case when "boundary" CFT's admit as symmetries 2D conformal transformations only. When more symmetries are allowed as for example the spin one currents $J_n^{(a)}$ generating local (say $SU(N)$) gauge transformations, the ranges of $c_q(N)$ are changed and the corresponding classical limits have different form [24] including now (eventually large) rank N of the gauge group.

In conclusion : the complete identification of the QFT_2 dual to NMG model requires (1) fur-

⁸some of them turns out to describe interesting 2D statistical models as for example the one of central charge $c = -\frac{22}{5}$ known as Lee-Yang edge singularity [5]

ther investigation of the properties of the correlation functions of 2D fields $\Phi_\sigma(x_i)$ and (2) better understanding of the NMG "corrections" (see eq.(7)) to 2D central charges introduced by the one loop \mathcal{K} -counter terms (1) that turns out to have classical limits similar to the ones of Liouville and m.m.central charges c_q^\pm .

This work has been partially supported by PRONEX project number 35885149/2006 from FAPES-CNPq (Brazil).

References

- [1] J.Maldacena, Adv.Theor.Math.Phys.2:231-252(1998).
- [2] E.Witten, Adv.Theor.Math.Phys.2:253(1998); S.S.Gubser,I.R.Klebanov and A.M.Polyakov, Phys.Lett.**428 B**(1998) 105.
- [3] A.A.Belavin ,A.M.Polyakov and A.B.Zamolodchikov, NPB{241}{1984}{333}.
- [4] A.B. Zamolodchikov, Sov. Phys. JETP Lett. **46**(1987)160; IJM PA{3}{1988}{743}; IJMPA{4}{1989}{4235};A.B. Zamolodchikov in "Integrable Systems in QFT and Statistical Mechanics", Adv. Stud. Pure Math. **19** (1989) 641
- [5] G.Mussardo , Statistical Field Theory, Oxford University Press Inc., New York, 2010.
- [6] P. Christe and G. Mussardo, NPB{330}{1990}{465} ; IJMPA{5}{1990}{4581}; G. Sotkov and C.J.Zhu,PLB{229}{1989}{391};V. Fateev and A.Zamolodchikov,IJMPA{5}{1990}{1025}; D. Bernard and A.LeClair, Commun. Math. Phys. **142** (1991) 99.
- [7] A.Zamolodchikov,Pisma JETP**25**,499(1977); A.Zamolodchikov and A.I.Zamolodchikov,Annals of Phys.(NY)120,253(1979); F.A.Smirnov,Form Factors in completely integrable models of Quantum Field Theory,Cambridge University Press,1995.
- [8] A.O.Gogolin,A.A.Nersesyan and A.M.Tsvelik,Bosonization and Strongly Correlated Systems,Cambridge University Press,1998.
- [9] E.A. Bergshoeff, O. Hohm and P.K. Townsend.,Phys. Rev. Lett.**102**, 201301(2009); Phy. Rev. **D79** 124042/2009.
- [10] S.S.Gubser,Class.Quant.Grav.17:1081-1092(2000).
- [11] J. de Boer, Fortsch. Phys. 49:339-358(2001), hep-th/0101026; E. Verlinde, H. Verlinde and J. de Boer , JHEP 0008:003(2000), hep-th/9912012.
- [12] D.Z. Freedman, S.S. Gubser, K.Pilch and N.P. Warner, Adv. Theor. Math. Phys. 3: 363-417(1999).
- [13] G.Clement,Class.Quant.Grav.**26**,105015(2009),Phys.Rev.Lett.**102**,201301(2009); E.Ayon-Beato,G.Giribet,M.Hassaine,JHEP0905,029 (2009);J.Oliva,D.Tempo and R.Trancoso,JHEP 0907:011(2009);H.Ahmedov and A.N.Aliev,The general type N solutions of New Massive Gravity,arxiv:1008.0303.
- [14] H.L.C. Louzada, U. Camara dS and G.M. Sotkov, Phys. Lett. **686 B** (2010) 268.
- [15] U. Camara dS and G.M. Sotkov, New Massive Gravity Domain Walls, arXiv:1008.2553.

- [16] O. Hohm and A. Tonni, JHEP 1004:093(2010) .
- [17] J.D.Brown and M.Henneaux, Commun.Math.Phys.**104**,207 (1986).
- [18] J.Cardy,Scaling and Renormalization in Statistical Physics,Cambridge Univ.Press,1996.
- [19] D.Z.Friedman,C.Nunez,M.Schnabl,K.Skenderis,Phys.Rev.**D69**,104 027(2004);
M.Cvetič,S.Griffies,S.J.Rey, Nucl. Phys **B381**1992301
- [20] M. Cvetič and H.H. Soleng, Phys. Rep. 282(1997) 159 and references therein.
- [21] P.Breitenlohner and D.Z.Freedman,PLB{115}{1982}{197};
Ann.Phys.144(1982)249.
- [22] A.B. Zamolodchikov, Sov. Phys. JETP Lett. **43**(1986)1731;
Sov.J.Nucl.Phys.**46**(1987),1090.
- [23] A.Sinha ,JHEP 1006:061(2010);R.Myers and A.Sinha,Phys.Rev.D82:046006,2010.
- [24] V.Fateev and S.Lukyanov, Sov.Sci.Rev.A.Phys.Vol.15 (1990),pp.1-117(see pg.18);
G.Sotkov and M.Stanishkov, NPB **356**,(1991),439.
- [25] A.B.Zamolodchikov and Al.B.Zamolodchikov, NPB{477}{1996}{577}, hep-th/9506136.